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Negativity and Vanishing of Microfunction Solution Sheaves at the Boundary

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Introduction. Let M be a real analytic manifold with a complexification X . Let V be a \mathbb{C}^\times -conic involutive submanifold of $\mathring{T}^*X (= T^*X \setminus X)$, and let \mathfrak{M} be a coherent \mathcal{E}_X -module with constant multiplicity along V . Moreover let Ω be an open subset of M with real analytic boundary $N = \partial\Omega$. The aim of this note is to give vanishing theorems for the cohomology groups of the complex $\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})$ where $\mathcal{C}_{\Omega|X}$ is the complex of microfunctions at the boundary introduced by P. Schapira [S].

The vanishing of the complex $\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_M)$ has been studied by M. Sato *et al.* [SKK], M. Kashiwara [K] and Kashiwara-Schapira [K-S2], and we study in this talk an analogous problem at the boundary.

Main Theorems. Let M, X, Ω and N be as in §1.1. Then we give

THEOREM 1. Let $p \in \mathring{T}_M^*X$ with $\pi_X(p) \in N$, and let $V = \{q \in \mathring{T}^*X; f(q) = 0\}$ be given by a homogeneous holomorphic function f satisfying the condition

$$(1) \quad \{f, f^c\}(p) < 0.$$

Assume that there exists a homogeneous holomorphic function ψ for which the following conditions (2), (3), (4), (5) are satisfied.

$$(2) \quad d\psi \wedge \omega_X \neq 0 \text{ at } p. \quad (\omega_X \text{ is the canonical 1-form of } T^*X.)$$

$$(3) \quad V \cap \overline{V} \subset \{\psi = 0\}.$$

$$(4) \quad \mathrm{Im} \, \psi|_{T_M^*X} = 0.$$

$$(5) \quad \pi_X^{-1}(\Omega) \cap T_M^*X \subset \{\psi > 0\} \text{ in a neighborhood of } p.$$

Let \mathfrak{M} be a coherent \mathcal{E}_X -module with constant multiplicity along V defined in a neighborhood of p . Then we have

$$H^0 \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})_p = 0.$$

THEOREM 2. Let p, V, Ω be as in Theorem 1. Let $W (\subset V)$ be a \mathbb{C}^\times -conic involutive variety in $\overset{\circ}{T}^*X$ through p with $q (\geq 1)$ negative eigenvalues of $\mathcal{L}_\Lambda(W)(p)$. Let \mathfrak{M} be a coherent \mathcal{E}_X -module with constant multiplicity along W defined in a neighborhood of p . Then we have

$$H^j \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})_p = 0 \quad (j < q).$$

Moreover if $\mathcal{L}_\Lambda(W)(p)$ is non-degenerate, then we have the vanishing of the left-hand side for $j \neq q$.

Next we give a generalization of Theorem 1.

THEOREM 3. Let $p \in \overset{\circ}{T}_M^*X$ with $\pi_X(p) \in N$, and let W be a \mathbb{C}^\times -conic involutive variety of codimension d with $p \in W$ and $\mathcal{L}_\Lambda(W)(p) < 0$. Assume that there exists a homogeneous holomorphic function ψ with the properties;

$$(6) \quad d\psi \wedge \omega_X \neq 0 \quad \text{at } p,$$

$$(7) \quad \mathrm{Im} \, \psi|_{T_M^*X} = 0,$$

$$(8) \quad W \cap \overline{W} \subset \{\psi = 0\},$$

$$(9) \quad \pi_X^{-1}(\Omega) \cap T_M^*X \subset \{\psi > 0\} \text{ in a neighborhood of } p.$$

Let \mathfrak{M} be a coherent \mathcal{E}_X -module with constant multiplicity along W . Then we have

$$H^j \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})_p = 0 \quad (j \neq d).$$

References

- [K] Kashiwara, M.: Vanishing theorems on the cohomologies of solution sheaves of systems of pseudo-differential equations. *Astérisque* **2-3** (1973), Soc. Math. de France, pp. 222-228.
- [K-S1] Kashiwara, M. and P. Schapira: Microlocal study of sheaves. *Astérisque* **128** (1985).
- [K-S2] —: A vanishing theorem for a class of systems with simple characteristics. *Invent. Math.* **82** (1985), pp. 579-592.
- [SKK] M. Sato, T. Kawai and M. Kashiwara: Hyperfunctions and pseudo-differential operators. *Lecture Notes in Math.* No. **287**, Springer, 1973, pp. 265-529.
- [S] —: Front d'onde analytique au bord II. Sémin. E.D.P., Ecole Polytechnique, Exposé 13, 1986.